

# Intrinsic Noise Sources, Noise Factor

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## References

**Basic:**  
Ott H. W., *Electromagnetic Compatibility Engineering*, Wiley, Hoboken, NJ, 2009

**Additional:**  
Williams T., *EMC for Product Designers*, Elsevier-Newnes, 4-th ed., Oxford, 2007

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## Source of illustrative materials

All the illustrative materials have been taken from:  
Ott H. W., *Electromagnetic Compatibility Engineering*, Wiley, Hoboken, NJ, 2009

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## Intrinsic Noise Sources

This lecture covers the two most important intrinsic noise sources: thermal noise and shot noise.

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## THERMAL NOISE

Thermal noise comes from thermal agitation of electrons **within a resistance**, and it sets a lower limit on the noise present in a circuit.  
The open-circuit rms noise voltage produced by a resistance is [Nyquist]

$$V_t = \sqrt{4KTBR}$$

where  
 K = Boltzmann's constant (1.38 · 10<sup>-23</sup> joules/K)  
 T = absolute temperature (K)  
 B = noise bandwidth (Hz)  
 R = resistance (Ω)

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## THERMAL NOISE

Thermal noise in a resistor can be represented in an equivalent circuit as a voltage source or a current source .

NORMAL SYMBOL	EQUIVALENT CIRCUIT (VOLTAGE SOURCE)	EQUIVALENT CIRCUIT (CURRENT SOURCE)

Electric circuit elements can produce thermal noise only if they can dissipate energy. Therefore, **a reactance cannot produce thermal noise.**

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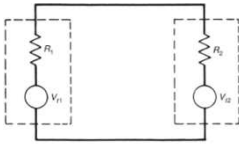
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### THERMAL NOISE

Let us consider the case when two resistors connected in parallel are equal in value, and maximum power transfer occurs between the resistors



We can write

$$P_{12} = P_{21} = P_n = \frac{V_i^2}{4R}$$

Substituting Nyquist dependence for  $V_i$  gives the "available noise power."

$$P_n = kTB$$

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### THERMAL NOISE

The thermal noise generated by any arbitrary network of passive elements is equal to the thermal noise that would be generated by **a resistance equal to the real part of the equivalent network impedance.**

This fact is useful for calculating the thermal noise of a complex passive network.

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### CHARACTERISTICS OF THERMAL NOISE

The frequency distribution of thermal noise power is uniform. Such noise—with a uniform power distribution with respect to frequency—is called "white noise".

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### CHARACTERISTICS OF THERMAL NOISE

The instantaneous value of thermal noise can only be defined in terms of probability and its amplitude has a Gaussian, or normal, distribution.

The probability density function for thermal noise:

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### EQUIVALENT NOISE BANDWIDTH

The noise bandwidth B is the voltage-gain-squared bandwidth of the system or circuit being considered. For any network transfer function, A(f) (expressed as a voltage or current ratio), an equivalent noise bandwidth exists with constant magnitude of transmission A<sub>0</sub> and bandwidth of

$$B = \frac{1}{|A_0|^2} \int_0^{\infty} |A(f)|^2 df$$

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### SHOT NOISE

Shot noise is associated with current flow across a potential barrier. It is caused by the fluctuation of current around an average value that results from the random emission of electrons (or holes).

The rms noise current is equal to

$$I_{sh} = \sqrt{2qI_{dc}B}$$

where  
 q = electron charge (1.6 · 10<sup>-19</sup> coulombs)  
 I<sub>dc</sub> = average dc current (A)  
 B = noise bandwidth (Hz)

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## Active Device Noise

Bipolar transistors, field effect transistors (FETs), and operational amplifiers (op-amps) have inherent noise-generation mechanisms.

The common methods of specifying device noise are  
 (1) noise factor and  
 (2) the use of a noise voltage and current model.

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## NOISE FACTOR

The noise factor (F) is a quantity that compares the noise performance of a device to that of an ideal (noiseless) device. It can be defined as

$$F = \frac{\text{Noise power output of actual device } (P_{no})}{\text{Noise power output of ideal device}}$$

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## NOISE FACTOR

The noise power output of an ideal device is due to the thermal noise power of the source resistance. The standard temperature for measuring the source noise power is 290K. The noise factor can be written as

$$F = \frac{\text{Noise power output of actual device } (P_{no})}{\text{Power output due to source noise}}$$

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### NOISE FACTOR

An equivalent definition of noise factor is the input signal-to-noise (S/N) ratio divided by the output signal-to-noise ratio

$$F = \frac{S_i/N_i}{S_o/N_o}$$

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### NOISE VOLTAGE AND CURRENT MODEL

The actual network can be modeled as a noise-free device with two noise generators,  $V_n$  and  $I_n$ , connected to the input side of a network:

NOISY NETWORK

IDEAL NOISELESS NETWORK

The use of these two noise generators plus their complex correlation coefficient completely characterizes the noise performance of the device.

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### NOISE VOLTAGE AND CURRENT MODEL

Typical curves of noise voltage and noise current:

NOISE VOLTAGE ( $\mu\text{V}/\sqrt{\text{Hz}}$ )

NOISE CURRENT ( $\text{pA}/\sqrt{\text{Hz}}$ )

FREQUENCY (kHz)

$[V_n/\sqrt{B}]^2$  or  $[I_n/\sqrt{B}]^2$

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### NOISE VOLTAGE AND CURRENT MODEL

Using these curves and the previously shown equivalent circuit, the total equivalent input noise voltage, signal-to-noise ratio, or noise factor for any circuit can be determined. This can be done for any source impedance, resistive or reactive, and across any frequency spectrum.

The representation of noise data in terms of the equivalent parameters  $V_n$  and  $I_n$  can be used for any device.

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### NOISE VOLTAGE AND CURRENT MODEL

Assuming no correlation between noise sources, the total equivalent input noise voltage of a device, which combines the effect of  $V_n$ ,  $I_n$ , and the thermal noise of the source, can be written as

$$V_{nt} = \sqrt{4kTBR_s + V_n^2 + (I_n R_s)^2}$$

where  $V_n$  and  $I_n$  are the noise voltage and noise current over the bandwidth B.

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### NOISE VOLTAGE AND CURRENT MODEL

The total equivalent input noise voltage per square root of bandwidth can be written as

$$\frac{V_{nt}}{\sqrt{B}} = \sqrt{4kTR_s + \left(\frac{V_n}{\sqrt{B}}\right)^2 + \left(\frac{I_n R_s}{\sqrt{B}}\right)^2}$$

The equivalent input noise voltage from the device noise only can be calculated by subtracting the thermal noise component from the last but one equation. The equivalent input device noise then becomes

$$V_{nd} = \sqrt{V_n^2 + (I_n R_s)^2}$$

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### CALCULATING NOISE FACTOR AND S/N RATIO FROM $V_n$ - $I_n$

Knowing the equivalent input noise voltage  $V_n$ , the current  $I_n$ , and the source resistance  $R_s$ , the noise factor can be calculated

$$F = 1 + \frac{1}{4kTB} \left( \frac{V_n^2}{R_s} + I_n^2 R_s \right)$$

where  $V_n$  and  $I_n$  are the equivalent input noise voltage and current over the bandwidth  $B$  of interest.

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### CALCULATING NOISE FACTOR AND S/N RATIO FROM $V_n$ - $I_n$

The value of  $R_s$  producing the minimum noise factor can be determined by differentiating the last equation with respect to  $R_s$ . The resulting  $R_s$  for minimum noise factor is

$$R_{so} = \frac{V_n}{I_n}$$

the minimum noise factor is

$$F_{min} = 1 + \frac{V_n I_n}{2kTB}$$

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### CALCULATING NOISE FACTOR AND S/N RATIO FROM $V_n$ - $I_n$

The output power signal-to-noise ratio is

$$\frac{S_o}{N_o} = \frac{(V_s)^2}{(V_n)^2 + (I_n R_s)^2 + 4kTB R_s}$$

where  $V_s$  is the input signal voltage.

For constant  $V_s$  maximum signal-to-noise ratio occurs when  $R_s = 0$ , and is

$$\left. \frac{S_o}{N_o} \right|_{max} = \left( \frac{V_s}{V_n} \right)^2$$

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### CALCULATING NOISE FACTOR AND S/N RATIO FROM $V_n$ - $I_n$

A plot of the total equivalent input noise voltage  $V_{nt}$  for a typical device

LOG OF NOISE VOLTAGE

LOG OF SOURCE RESISTANCE, ( $R_s$ )

TOTAL NOISE  $V_{nt}$

NOISE ALMOST CONSTANT ACROSS THIS RANGE OF  $R_s$  VALUES

MINIMUM TOTAL NOISE  $I_n R_s$

THERMAL NOISE IN  $R_s$   $\sqrt{kTB}$

$R_s = 0$  FOR MAXIMUM S/N RATIO

$R_s$  FOR MAXIMUM NOISE FACTOR

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### OPTIMUM SOURCE RESISTANCE

For optimum noise performance, the lowest possible source impedance should be used.

Once this is decided, noise performance can be improved by transformer coupling this source to match the impedance  $R_s = V_n/I_n$ .

The improvement in signal-to-noise ratio resulting from transformer coupling

$$SNI = \frac{(F) \text{ without transformer}}{(F) \text{ with transformer}}$$

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### NOISE FACTOR OF CASCADED STAGES

Networks in cascade

NETWORK 1 GAIN =  $G_1$  NOISE FACTOR =  $F_1$

NETWORK 2 GAIN =  $G_2$  NOISE FACTOR =  $F_2$

NETWORK  $m$  GAIN =  $G_m$  NOISE FACTOR =  $F_m$

The overall noise factor of a series of networks connected in cascade is

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_m - 1}{G_1 G_2 \dots G_{m-1}}$$

where  $F_1$  and  $G_1$  are the noise factor and available power gain of the first stage,  $F_2$ ,  $G_2$  are those of the second stage, and so on.

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### NOISE TEMPERATURE

The equivalent input noise temperature of a circuit can be defined as the increase in source resistance temperature necessary to produce the observed noise power at the output of the noiseless circuit. The idea is illustrated below

Amplifier with noise

NOISY AMPLIFIER  
GAIN = G  
REFERENCE TEMPERATURE  $T_0 = 290\text{K}$   
 $P_n = G^2 k T_0 B F$

Source resistance temperature increased to account for amplifier noise

NOISELESS AMPLIFIER  
GAIN = G  
 $P_n = G^2 k T_e B$

The standard reference temperature  $T_0$  for noise temperature is 290K.

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### NOISE TEMPERATURE

The equivalent input noise temperature is related to the noise factor F by

$$T_e = 290(F - 1)$$

In terms of the equivalent input noise voltage and current ( $V_n - I_n$ ) the noise temperature can be written as

$$T_e = \frac{V_n^2 + (I_n R_s)^2}{4kBR_s}$$

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### BIPOLAR TRANSISTOR NOISE

The noise figure versus frequency for a typical bipolar transistor is shown in the figure below

NOISE FIGURE (dB)

LOG OF FREQUENCY

3 dB/OCTAVE

6 dB/OCTAVE

MID-BAND

PROPORTIONAL TO  $1/f$

PROPORTIONAL TO  $1/f$

PROPORTIONAL TO  $(f/f_0)^2$

$f_0 = f_c / \sqrt{1 - \alpha_0}$

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### NOISE IN OPERATIONAL AMPLIFIERS vs. DISCRETE TRANSISTOR AMPLIFIERS

In general, operational amplifiers are inherently higher noise devices than discrete transistor amplifiers.

The graph plots Total Equivalent Input Noise Voltage  $V_n$  (V/√Hz) on the y-axis (log scale from  $10^{-10}$  to  $10^{-5}$ ) against Source Resistance  $R_s$  (Ω) on the x-axis (log scale from 1 to 10M). A dashed line represents Thermal Noise in  $R_s$ . The IC Op-Amp curve is the highest, followed by the Junction FET, and the Bipolar Transistor curve is the lowest. All curves show an increase in noise with increasing source resistance.

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### Thank you for your attention

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