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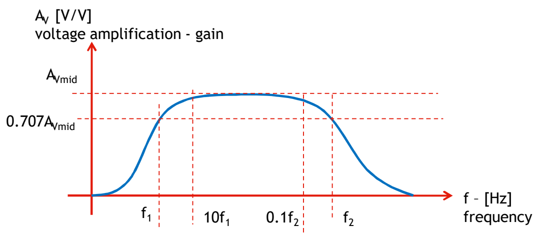
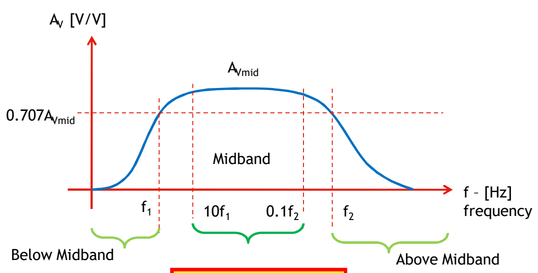
Amplifiers principles, frequency effects & some other parameters

Jerzy S. Witkowski

Response of an AC amp

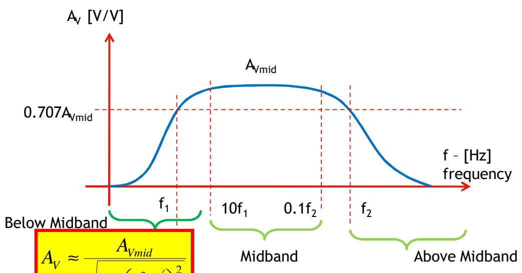


Low/Mid/High frequency



$$A_V \approx A_{Vmid} \approx const$$

Low/Mid/High frequency



$$A_V \approx \frac{A_{Vmid}}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}$$

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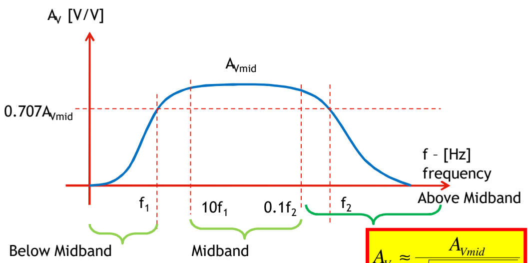
Low/Mid/High frequency



$$A_V \approx \frac{A_{Vmid}}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}}$$

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Low/Mid/High frequency

$$A_V \approx \frac{A_{Vmid}}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}\sqrt{1+\left(\frac{f}{f_2}\right)^2}} \Leftrightarrow \frac{A_{Vmid}}{\left(1+\frac{1}{R_1C_1s}\right)\left(1+R_2C_2s\right)}$$

Assumption:
One dominant capacitor is producing lower cutoff frequency
One dominant capacitor is producing high cutoff frequency

-*"first order poles"*

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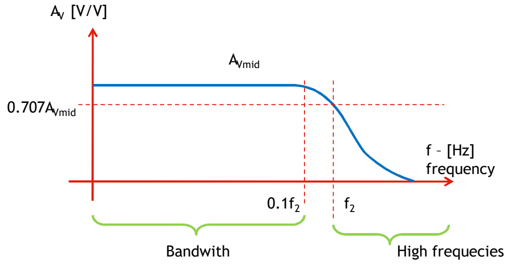
DC amplifier



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DC amplifier

$$A_V \approx \frac{A_{Vmid}}{\sqrt{1+\left(f/f_2\right)^2}} \Leftrightarrow \frac{A_{Vmid}}{(1+R_2C_2s)}$$

Assumption:
One dominant capacitor determines the upper cutoff frequency

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Decibel power gain
review of logarithms

$$x = 10^y \Leftrightarrow y = \log_{10}(x) = \log(x)$$

$\log(1) = \log(10^0) = 0$	$\log(1) = 0$
$\log(10) = 1$	$\log(0.1) = -1$
$\log(100) = 2$	$\log(0.01) = -2$
$\log(1000) = 3$	$\log(0.001) = -3$

$$\log(ab) = \log(a) + \log(b)$$

$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$

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Decibel power gain - $A_{p(dB)}$

power gain:

$$A_P = \frac{P_{out}}{P_{in}}$$

power gain in dB:

$$A_{P(dB)} = 10\log\left(\frac{P_{out}}{P_{in}}\right) = 10\log(A_P)$$

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Decibel power gain

- important numbers

A_P (factor)		$A_{P(dB)}$
=10*10	100	20
=10*2*2	40	+16
=10*2	20	+13
10	10	+10
=2*2	4	+6
2	2	+3
1/2	0.5	-3
=(1/2) * (1/2)	0.25	-6
=(1/10)	0.1	-10
=(1/2)*(1/10)	0.05	-13
=(1/2)*(1/2)*(1/10)	0.025	-16
=(1/10)*(1/10)	0.01	-20

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Decibel power gain

- important numbers

	A_P (factor)		$A_{P(dB)}$
amplification	10	10	+10
	2	2	+3
attenuation	1/2	0.5	-3
	1/10	0.1	-10

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Decibel voltage gain

voltage gain:

$$A_V = \frac{U_{out}}{U_{in}}$$

voltage gain in dB:

$$A_{P(dB)} = 20\log\left(\frac{U_{out}}{U_{in}}\right) = 20\log(A_V)$$

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Decibel voltage gain
- important numbers

A_V (factor)		$A_{V(dB)}$
=10*10	100	40
10	10	+20
=2*2	4	+12
2	2	+6
=1.41	√2	+3
=0.707	1/√2	-3
1/2	0.5	-6
=(1/2) * (1/2)	0.25	-6
=(1/10)	0.1	-20
=(1/10)*(1/10)	0.01	-40

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Decibel voltage gain
- important numbers

A_V (factor)		$A_{V(dB)}$
10	10	+20
2	2	+6
=1.41	√2	+3
=0.707	1/√2	-3
1/2	0.5	-6
1/10	0.1	-20

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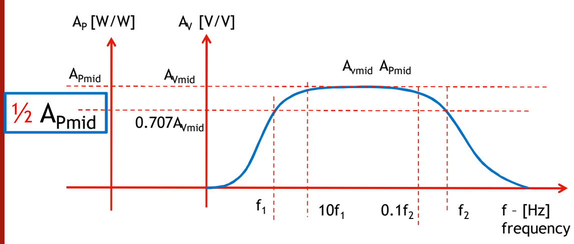
Response of an AC amp



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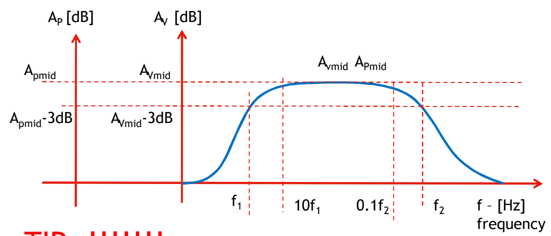
Response of an AC amp



TIP :!!!!!!!

-3dB= power decreases twice and voltage √2 times

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Cutoff frequencies

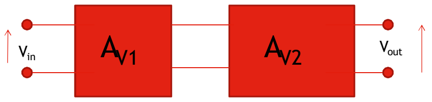
Cutoff frequencies

Cutoff frequencies

TIP :!!!!!!!

-3dB= power decreases twice
and voltage decreases√2≈0.707 times

[dB] vs.[V/V]



$$A_V = \frac{U_{out}}{U_{in}} = A_{V1} \cdot A_{V2}$$

$$A_{V(dB)} = A_{V1(dB)} + A_{V2(dB)}$$

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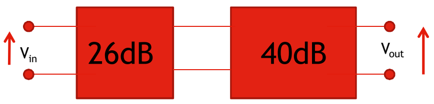
[dB] vs.[V/V] an example



$$A_{V(dB)} = A_{V1(dB)} + A_{V2(dB)} = 26 + 40 = 66dB$$

$$66dB = 6dB + 20dB + 40dB$$

$$66dB \rightarrow \times 2 \times 10 \times 100 = 2000[V/V]$$

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Decibels above a REFERENCE dBm, dBμ, dBV…..

$$P[dBm] = 10\log\left(\frac{P[W]}{1mW}\right)$$

$$U[dB\mu] = 20\log\left(\frac{U[V]}{1\mu V}\right)$$

$$U[dBV] = 20\log\left(\frac{U[V]}{1V}\right)$$

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Some more dB

- dBi (isotropic antenna)
- dBc (carrir)
- dBA

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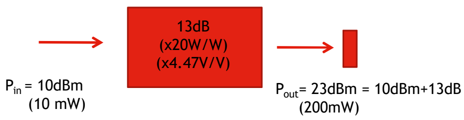
dBm an exampl

$$10dBm \oplus 23dBm =$$
$$= 10mW + 200mW = 210mW = 23.2dBm$$



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Summary

- cutoff frequecies ?
- dB for voltage and power gains
- dB vs. factors (3,6,10,20 dB)
- dBm, dBu, dBV ?

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Bode plots

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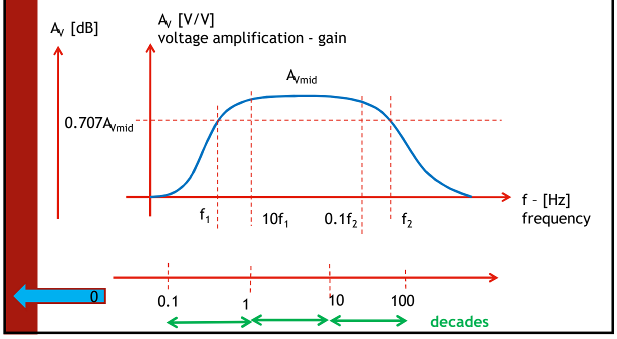
Bode plots -logarithmic scales



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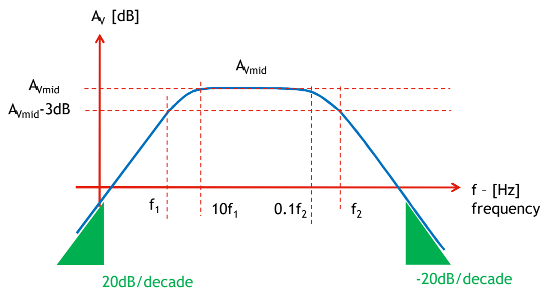
Bode plot



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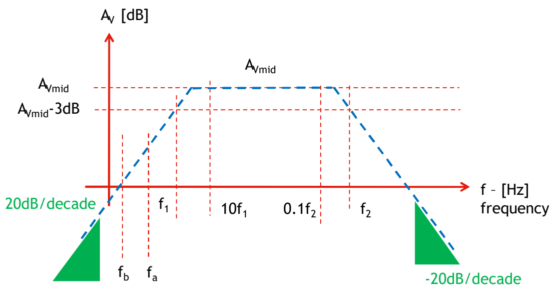
Bode plot - low f



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Bode plot
- 20dB/decade ???

$$A_V \approx \frac{A_{Vmid}}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}$$

$$A_{V(dB)}/decade = 20\log\frac{A_V(f_a)}{A_V(f_b)} = 20\log\frac{\sqrt{1+\left(\frac{f_1}{f_a}\right)^2}}{\sqrt{1+\left(\frac{f_1}{f_b}\right)^2}} = 20\log\sqrt{\frac{f_a{}^2+f_1^2}{f_b{}^2+f_1^2}\frac{f_b{}^2}{f_a{}^2}}$$

$$1\,decade = \frac{f_b}{f_a} = 10$$

$$f_a, f_b \ll f_1$$

$$A_{V(dB)}/decade = 20\log\sqrt{\frac{f_a{}^2+f_1^2}{f_b{}^2+f_1^2}}\sqrt{\frac{f_b{}^2}{f_a{}^2}} \approx 20\log\frac{f_b}{f_a} = 20\log(0.1) = +20[dB/dec]$$

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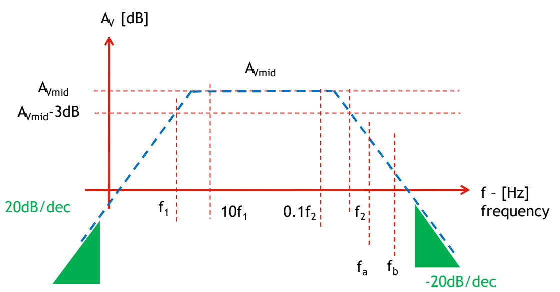
Bode plot - high f



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Bode plot - 20dB/decade ???

$$A_V \approx \frac{A_{Vmid}}{\sqrt{1+\left(f/f_2\right)^2}}$$

$$A_{V(dB)}/decade = 20\log\frac{A_V(f_a)}{A_V(f_b)} = 20\log\frac{\sqrt{1+\left(f_a/f_2\right)^2}}{\sqrt{1+\left(f_b/f_2\right)^2}} = 20\log\sqrt{\frac{f_a^{\,2}+f_2^{\,2}}{f_b^{\,2}+f_2^{\,2}}}$$

$$1\,decade = f_a/f_b = 0.1$$

$$f_a, f_b >> f_2$$

$$A_{V(dB)}/decade = 20\log\sqrt{\frac{f_a^{\,2}+f_2^{\,2}}{f_b^{\,2}+f_2^{\,2}}} \approx 20\log\frac{f_a}{f_b} = 20\log(0.1) = -20[dB/dec]$$

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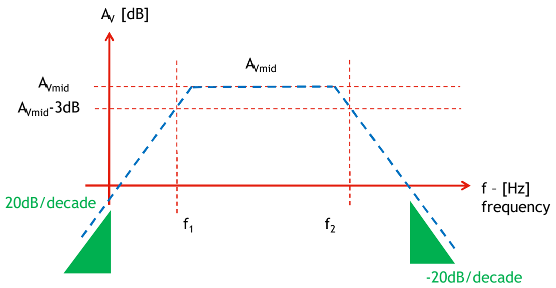
Bode plot + simplified frequency response



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20dB/dec = 6dB/oct

$$A_{V(dB)}/decade = 20\log\sqrt{\frac{f_a^{\,2}+f_2^{\,2}}{f_b^{\,2}+f_2^{\,2}}} \approx 20\log\frac{f_a}{f_b} = 20\log(0.1) = -20[dB/dec]$$

$$A_{V(dB)}/oct = 20\log\sqrt{\frac{f_a^{\,2}+f_2^{\,2}}{f_b^{\,2}+f_2^{\,2}}} \approx 20\log\frac{f_a}{f_b} = 20\log(\frac{1}{2}) \approx -6[dB/oct]$$

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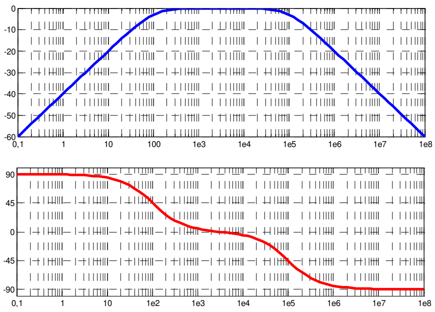
$$\frac{1}{\left(1+\frac{1}{R_1C_1s}\right)\left(1+R_2C_2s\right)}$$

f_1=100Hz ; f_2=100kHz



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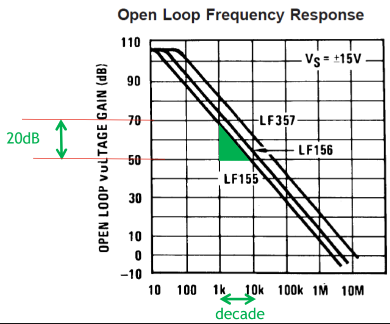
Bode plot op amp example



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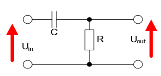
20dB/decade slope derivation

$$A_V \approx \frac{A_{Vmid}}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}$$



$$A_V=\frac{U_{out}}{U_{in}}=\frac{R}{R+\frac{1}{j\omega C}}=\frac{1}{1+\frac{1}{j\omega CR}}=\frac{1}{1+\frac{\frac{1}{RC}}{j\omega}}=\frac{1}{1+\frac{\omega_1}{j\omega}}=\frac{1}{1+\frac{f_1}{jf}}=$$

$$=\frac{1}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}e^{arctg\left(\frac{f_1}{f}\right)}=\left\langle\frac{1}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}};\varphi=artg\left(\frac{f_1}{f}\right)\right\rangle$$

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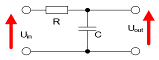
20dB/decade slope derivation

$$A_V \approx \frac{A_{Vmid}}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}}$$



$$f_2 = \frac{1}{2\pi RC}$$

$$A_V = \frac{U_{out}}{U_{in}} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega CR} = \frac{1}{1+j\omega\frac{1}{\frac{1}{RC}}} = \frac{1}{1+\frac{j\omega}{\omega_2}} = \frac{1}{1+j\frac{f}{f_2}} =$$

$$= \frac{1}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}}\, e^{-actg\left(\frac{f}{f_2}\right)} = \left\langle \frac{1}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}}; \varphi = -artg\left(\frac{f}{f_2}\right)\right\rangle$$

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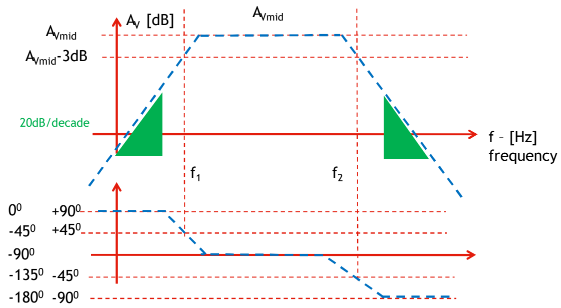
Bode plots - phase siplified plot



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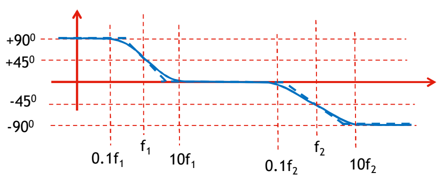
Bode plot -phase - real plot



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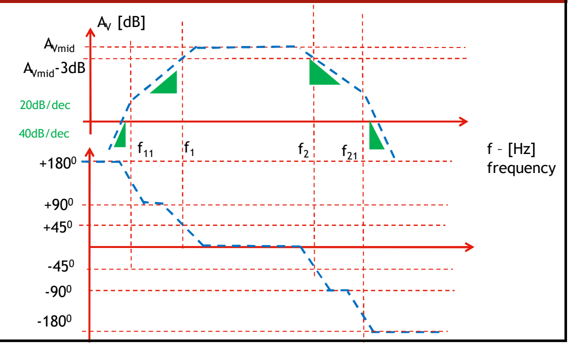
Higher order Bode plots
simplified plot



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Slope vs. phase

frequency →

Slope [dB/dec]	+60	+40	+20	0	-20	-40	-60
Phase [deg]	+270	+180	+90	0	-90	-180	-270
Transfer function	~s^3	~s^2	~s	const	~1/s	~$1/s^2$	~$1/s^3$

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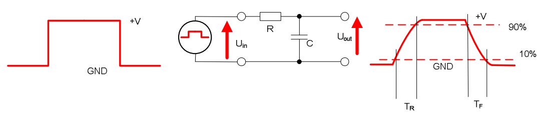
rise time vs. bandwith



$$\frac{U_{out}}{+V} = \left(1 - e^{-\frac{t}{RC}}\right)$$

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rise time vs. bandwith

$$\left.\begin{aligned} 0.9 &= \left(1-e^{-\frac{t_2}{RC}}\right) \Rightarrow -\frac{t_2}{RC} = \ln(0.1) \\ 0.9 &= \left(1-e^{-\frac{t_1}{RC}}\right) \Rightarrow -\frac{t_1}{RC} = \ln(0.9) \end{aligned}\right\} T_R = t_2 - t_1 = RC\ln(9) \approx 2.2RC$$

$$f_2 = \frac{1}{2\pi RC}$$

$$T_R = \frac{0.35}{f_2} = T_F$$

$$0.35 = \frac{2\pi}{\ln(90\% / 10\%)}$$

Assumption:
One dominant capacitor is producing high cutoff frequency

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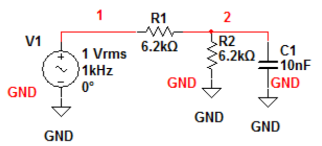
Simulink analysis - low pass filter



A(0Hz)=-6dB

f_2=5.134Hz

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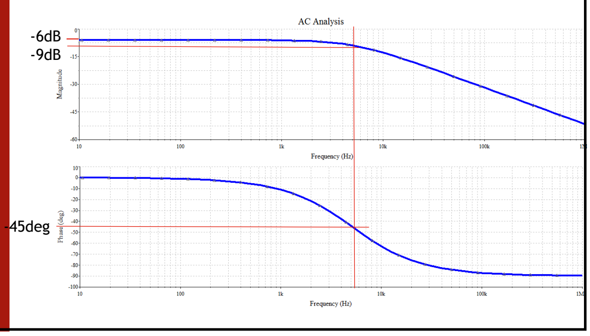
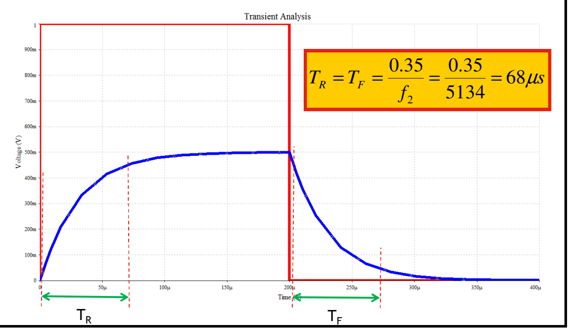
Simulink analysis – l.p. result

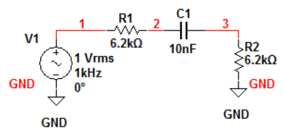


Fall & Rise time - example



$$T_R = T_F = \frac{0.35}{f_2} = \frac{0.35}{5134} = 68\mu s$$

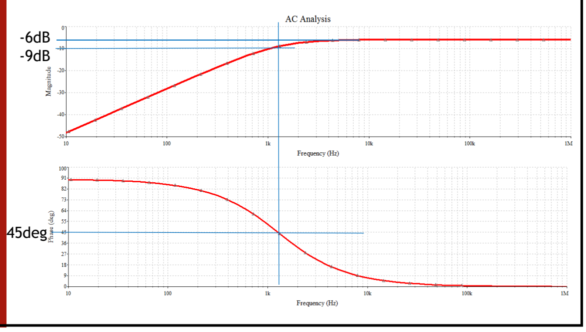
Simulink analysis - high pass filter



$A(\infty Hz) = -6dB$

$f_1 = 1.283Hz$

Simulink analysis - h.p. result



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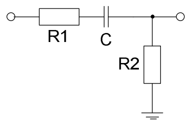
A few examples



R1	C	R2	f1	Av(f=∞)
5k1	100n	10k	105Hz	-3.6dB
10k	100n	5k1	105Hz	-9.5dB
10k	220n	5k1	50Hz	-9.5dB
10k	100n	51k	26Hz	-1.6dB

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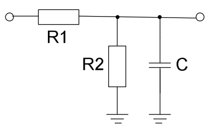
A few examples



R1	R2	C	f2	Av(f=0)
10k	5k1	1n	105Hz	-9.5dB
5k1	10k	1n	105Hz	-3.6dB
5k1	10k	2n2	53Hz	-3.6dB
k51	10k	1n	26Hz	-0.4dB

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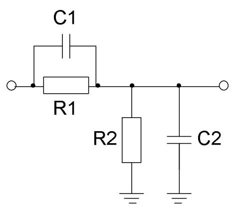
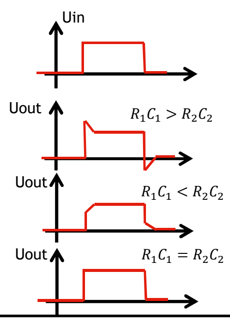
Compensated divider





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Bode plot - conclusions

- the absolute value and the phase of transfer function are correlated (exceptions exists)
- for circuits with one dominant capacitor (first order t. function) the gain rise/fall (low/high frequency) 20dB/dec =6dB/oct
- for cutoff frequencies (-/+3dB below maximum gain) the pase is +/- 45deg
- when the gain rise/fall 20dB/dec the phase is +/-90deg
- compensated divider- perfect pulse response- flat frequency response

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Problems (Amplifiers principles, Bode plots)

- What is dB ?
- dB (±3,±6,±10,±20) vs. [V/V] and [W/W]
- What is dBμ, dBm, dBV ?
- What is idea of Bode plots ?
- Draw Bode plot for low-/high- pass first order filter.
- What is the idea of compensated divider ?

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Power amplification

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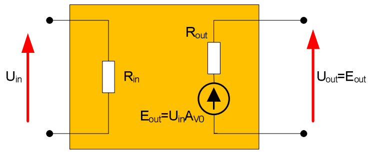
Voltage amplfication unloaded amp



$$A_V = A_{V0}$$

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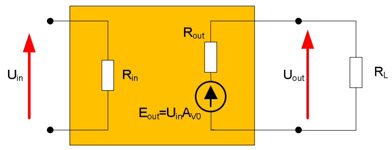
Voltage amplfication loaded amp



$$A_V = \frac{U_{out}}{U_{in}} = \frac{E_{out}}{U_{in}}\frac{U_{out}}{E_{out}} = A_{V0}\frac{R_L}{R_L + R_{out}}$$

TIP:
This A_V usually announced in advertisements

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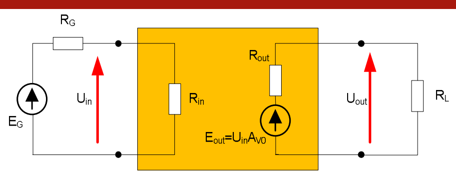
Effective voltage amplification -unmatched circuit



$$A_V = \frac{U_{out}}{U_{in}} = \frac{E_{out}}{U_{in}}\frac{U_{out}}{E_{out}} = A_{V0}\frac{R_L}{R_L + R_{out}}$$

$$A_{Veff} \overset{def}{=} \frac{U_{out}}{E_G} = \frac{U_{in}}{E_G}\frac{E_{out}}{U_{in}}\frac{U_{out}}{E_{out}} = \underbrace{\frac{R_{in}}{R_G + R_{in}}}_{\gamma}\underbrace{A_{V0}\frac{R_L}{R_L + R_{out}}}_{A_V} = \gamma A_V$$

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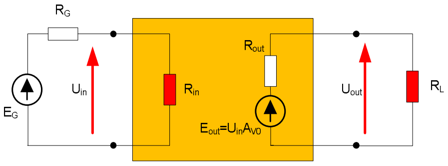
Power amplification



$$P_{in} = \frac{U_{in}^2}{R_{in}} \qquad P_{out} = \frac{U_{out}^2}{R_L}$$

$$A_P = \frac{P_{out}}{P_{in}} = \left(\frac{U_{out}}{U_{in}}\right)^2 \frac{R_{in}}{R_L} = (A_V)^2 \frac{R_{in}}{R_L}$$

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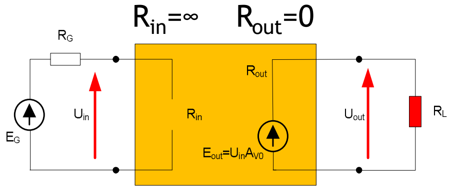
Voltage vs. power amplification in dB case $R_{out}=0$; $R_{in}=\infty$

$R_{in}=\infty$ $R_{out}=0$



$$A_P = \frac{P_{out}}{P_{in}} = \left(\frac{U_{out}}{U_{in}}\right)^2 \frac{R_{in}}{R_L} = (A_V)^2 \frac{R_{in}}{R_L} = \infty$$

TIP:
Operational amplifier has infinite power gain
-input power is zero !!!!

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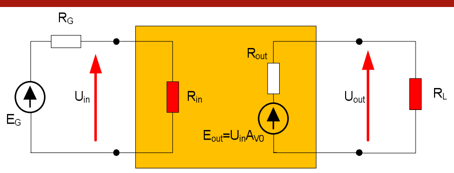
Power amplification



$$P_{in} = \frac{U_{in}^2}{R_{in}} \qquad P_{out} = \frac{U_{out}^2}{R_L}$$

$$A_P = \frac{P_{out}}{P_{in}} = \left(\frac{U_{out}}{U_{in}}\right)^2 \frac{R_{in}}{R_L} = (A_V)^2 \frac{R_{in}}{R_L}$$

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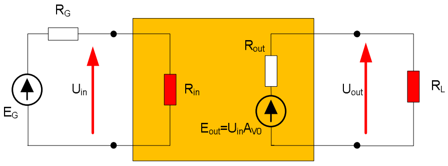
Voltage vs. power amplification in dB



$$A_P = \frac{P_{out}}{P_{in}} = \left(\frac{U_{out}}{U_{in}}\right)^2 \frac{R_{in}}{R_L} = (A_V)^2 \frac{R_{in}}{R_L}$$

$$A_{P(dB)} = 10\log\left(\frac{P_{out}}{P_{in}}\right) = 10\log\left(\left(\frac{U_{out}}{U_{in}}\right)^2 \frac{R_{in}}{R_L}\right) = 20\log\left(\frac{U_{out}}{U_{in}}\right) + 10\log\left(\frac{R_{in}}{R_L}\right) = A_{V(dB)} + 10\log\left(\frac{R_{in}}{R_L}\right)$$

$$A_{P(dB)} = A_{V(dB)} + 10\log\left(\frac{R_{in}}{R_L}\right)$$

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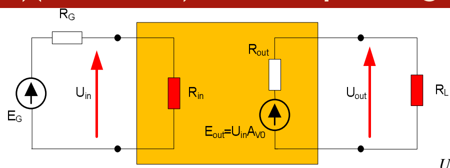
Avaliable input/output power --)(maximum) avilable power gain



$$P_{in\max} = \frac{E_G^2}{4R_G} \qquad P_{out\max} = \frac{U_{out}^2}{4R_{out}}$$

Available power gain= (Available output power)/(Available input power)

$$A_{P-available} = \frac{P_{out\max}}{P_{in\max}} = \left(\frac{U_{out}}{E_G}\right)^2 \frac{R_G}{R_{out}} = (A_{Veff})^2 \frac{R_G}{R_{out}} = \left(\frac{R_{in}}{R_G + R_{in}} A_{V0} \frac{R_L}{R_{out} + R_L}\right)^2 \frac{R_G}{R_{out}}$$

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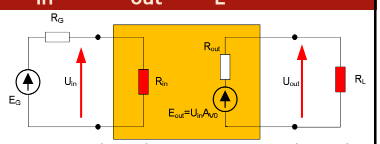
---- for matched circuits $R_G = R_{in}$ & $R_{out} = R_L$



$$\text{Available power gain} = \frac{\text{Available output power}}{\text{Available input power}}$$

$$P_{in\max} = \frac{E_G^2}{4R_G} = \frac{U_{in}^2}{R_{in}} \qquad P_{out\max} = \frac{E_{out}^2}{4R_{out}} = \frac{U_{out}^2}{R_L}$$

$$A_{P-available} = \frac{P_{out\max}}{P_{in\max}} = \left(\frac{E_{out}}{E_G}\right)^2 \frac{R_G}{R_{out}} = \left(\frac{U_{out}}{U_{in}}\right)^2 \frac{R_{in}}{R_L}$$

$$A_{P-availabla} = A_{P(dB)} = A_{V(dB)} + 10\log\left(\frac{R_{in}}{R_L}\right) = A_{V(dB)} + 10\log\left(\frac{R_G}{R_L}\right)$$

$$R_G = R_{in} \qquad R_{out} = R_L$$

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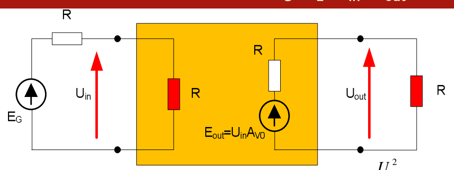
---- for matched circuits

- the most offten case $R_G=R_L=R_{in}=R_{out}=R$



$$P_{in}=\frac{U_{in}^2}{R}=P_{in\max}=\frac{E_G^2}{4R} \qquad P_{out}=\frac{U_{out}^2}{R}=P_{out\max}=\frac{E_{out}^2}{4R}$$

Available power gain= Available output power)/(Available input power)

$$A_{P-available}=\frac{P_{out\max}}{P_{in\max}}=\left(\frac{E_{out}}{E_G}\right)^2=\frac{P_{out}}{P_{in}}=\left(\frac{U_{out}}{U_{in}}\right)^2=\left(A_{V0}\frac{1}{2}\right)^2=A_V=4\left(\frac{U_{out}}{E_G}\right)^2=4\left(A_{Veff}\right)^2$$

$$A_{P(dB)}=A_{P(dB)-available}=A_{V(dB)}=A_{Veff(dB)}+6dB$$

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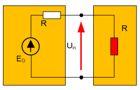
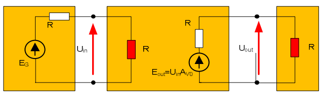
---- for matched circuits another approach

- the most offten case $R_G=R_L=R_{in}=R_{out}=R$



$$U_{in}=\frac{E_G}{2} \qquad P_1=\frac{E_G^2}{4R}$$



$$U_{out}=\frac{E_G}{2}\left(A_{V0}\frac{1}{2}\right) \qquad P_2=\frac{E_G^2A_{V0}^2}{16R}$$

$$A_V=\frac{U_{out}}{U_{in}}=\frac{A_{V0}}{2} \qquad A_P=\frac{P_2}{P_1}=\left(\frac{A_{V0}}{2}\right)^2$$

$$A_P[dB]=A_V[dB]$$

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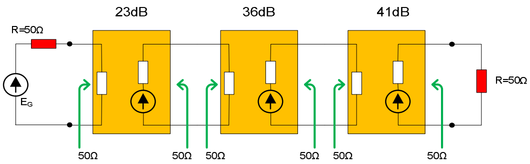
50Ohm system

$$P_{dB}=A_{VdB}=100dB; \quad A_P=10^{100/10}=10^{10}; \quad A_V=10^{100/20}=100000$$



$$P_{dB}=A_{VdB}=23dB \quad A_P=10^{23/10}=200 \quad A_V=10^{23/20}=14.1=\sqrt{200}$$

$$P_{dB}=A_{VdB}=36dB \quad A_P=10^{36/10}=3981 \quad A_V=10^{26/20}=63.1$$

$$P_{dB}=A_{VdB}=41dB \quad A_P=10^{41/10}=12589 \quad A_V=10^{41/20}=112$$

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dBm vs. dBμ

$$P_{dBm} = 10\log\frac{P_W}{1mW}$$

$$P_W = 1mW \cdot 10^{P_{dBm}/10} = \frac{U^2}{R}$$

$$U = \sqrt{1mW \cdot R \cdot 10^{P_{dBm}/10}}$$

$$U_{dB\mu} = 20\log\frac{\sqrt{1mW \cdot R \cdot 10^{P_{dBm}/10}}}{1\mu V} = 20\log\frac{\sqrt{1mW \cdot R}}{1\mu V} + 20\log\sqrt{10^{P_{dBm}/10}}$$

$$U_{dB\mu} = 10\log\left(10^9 R\right) + P_{dBm} = 80 + 10\log\left(10R\right) + P_{dBm}$$

$$\boxed{U_{dB\mu} = 107 + P_{dBm}}$$

for R=50Ω

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Problems (power amplification)

- voltage gain vs. effective voltage gain
- Voltage vs. power amplification in dB
- available power gain ?
- available power gain for matched systems ?