Analog multipliers

General

$$u_{out} = k_m u_x u_y = \frac{u_x u_y}{E_R}$$

u_x, u_y – input voltages,

$k_m = 1/E_R$ – scale constant,

E_R – standarization voltage e.g. +10V, -10V.

General types of input ranges

- one-quadrant – u_x and $u_y > 0$,
- two-quadrant – $u_x > 0$ and $u_y >=< 0$,
- four-quadrant – u_x and $u_y >=< 0$,

Types - principles

- Modulation (+voltage variable resistors),
- logarithmic amplifiers,
- squarer ,
- transconductance.

Errors

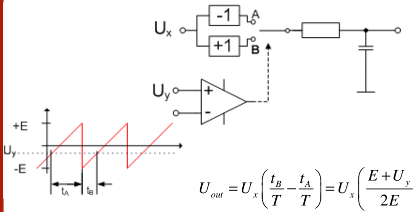
$$u_{out} = \frac{u_x u_y}{E_R} + \Delta = \frac{u_x u_y}{E_R}(1+\delta_0)$$

Δ, δ_0 – absolute and relative error.

Parameters:

- f_{3dB} – cut off frequency ????,
- f_{α} – 1% error frequency of amplitude,
- f_{φ} – 1% error of phase,
- SR – slew rate

Pulse with modulation multiplier

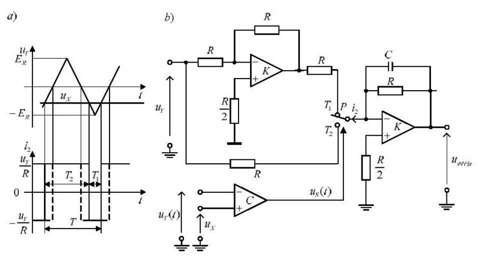


$$U_{out} = U_x\left(\frac{t_B}{T} - \frac{t_A}{T}\right)$$

$$U_{out} = U_x\left(\frac{t_B}{T} - \frac{t_A}{T}\right) = U_x\left(\frac{E+U_y}{2E} - \frac{E-U_y}{2E}\right) = \frac{U_x \cdot U_y}{E}$$

$$\frac{t_B}{T} = \frac{E+U_y}{2E}; \quad \frac{t_A}{T} = \frac{E-U_y}{2E};$$

Pulse with modulation multiplier practical example



Pulse with modulation multiplier

$$u_S(t) = \begin{cases} U_{S\max} & \text{dla} \quad u_x > u_T \\ U_{S\min} & \phantom{\text{dla}} \quad u_x < u_T \end{cases}$$

$$\frac{u_x}{E_R} = \frac{2T_1}{T} - 1$$

$$u_{out} = \left(\frac{2T_1}{T} - 1\right) u_y = \frac{u_x u_y}{E_R}$$

Pulse with modulation multiplier properties

High accuracy: δ_0 = (0.01…0.1 %), f_{3dB} = 1kHz, SR <0.7V/ms.

Limitations:

- Turn on and off of the switch should by much smaller than the period,
- frequency of low-pass filter much smaller than the working frequency.

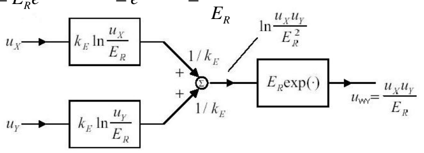
General:

- High accuracy
- Small bandwith
- Complicated - expensive

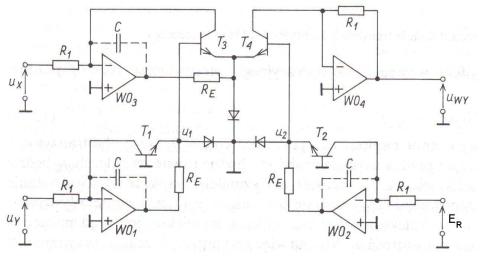
Logarithmic - exponential amplifiers

$$u_{WY} = E_R e^{\left(\ln\frac{u_x}{E_R}+\ln\frac{u_y}{E_R}\right)} = e^{\left(\ln\frac{u_x u_y}{E_R E_R}\right)} = \frac{u_x u_y}{E_R}$$

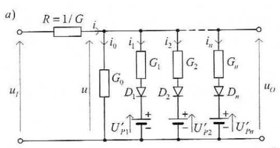
dla $u_x, u_y > 0$



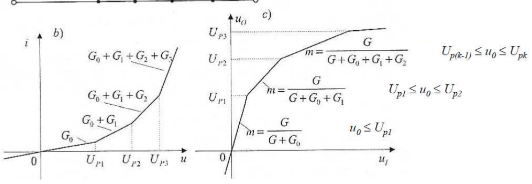
Logarithmic - exponential amplifiers example



Logarithmic - exponential amplifiers arbitrary function amplifier



Diodes are working as switches:
$U_k - iR > U_{pk}$

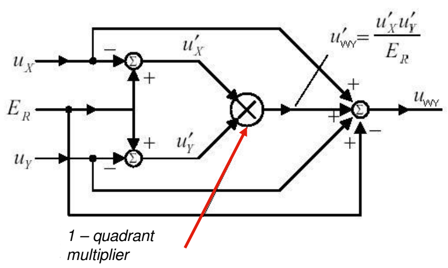


Multipier with logarithmic - exponential amplifiers

$$u_{wy} = \exp(\ln u_x + \ln u_y - \ln E_R) = \frac{u_x u_y}{E_R} \quad \text{dla} \quad u_x, u_y, E_R > 0$$

One-quadrant multiplier.

Logarithmic - exponential amplifiers 4- quadrant extention



Logarithmic - exponential amplifiers 4- quadrant extention

for $E_R > 0$. $\quad u'_{out} = \frac{u'_x u'_y}{E_R}$

where:

$$u'_x = E_R - u_x > 0$$
$$u'_y = E_R - u_y > 0$$
$$u'_{out} = \frac{(E_R - u_x)(E_R - u_y)}{E_R} = E_R - u_x - u_y + \frac{u_x u_y}{E_R}$$

finaly:

$$u_{out} = u'_{out} - E_R + u_x + u_y = \frac{u_x u_y}{E_R}$$

Układy mnożące wykorzystujące operacje: logarytmiczną i wykładniczą

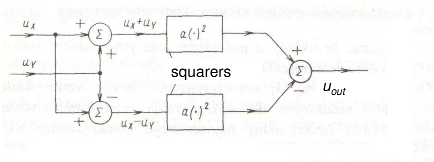
Input voltage range: ±10V

Accuracy (δ_0 = 0.1…0.5 %),

Bandwidth up to 250kHz

SR < 0.5 V/µs.

Multiplier with squarer



$$u_{out} = a\left(u_x + u_y\right)^2 - a\left(u_x - u_y\right)^2 = 4au_xu_y$$

Multiplier with squarer

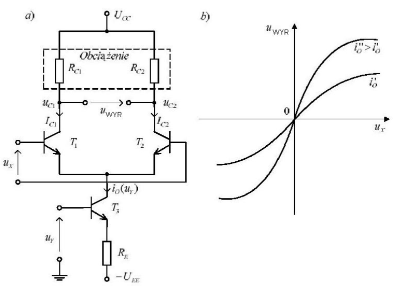
Squarer:

- arbitrary function amplifier (linear approximation),
- FET or MOSFET (input characteristic)

Properties:

- Error 0.5%,
- SR<3V/us
- f_{3dB} <2 MHz

Transconductance multiplier - two-quadrant



Transconductance multiplier

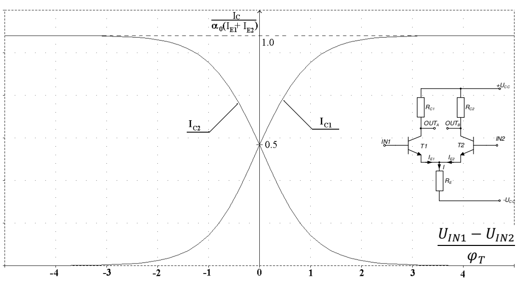
$$i_0(u_y) = I_0 + g_m u_y$$

$$u_{out} = i_0 R_C tgh \frac{u_x}{2\varphi_T}$$

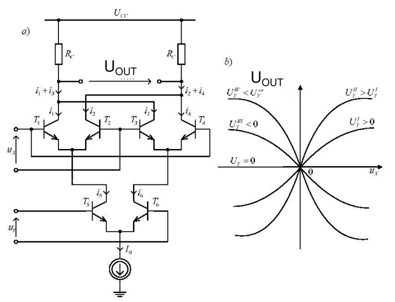
$$tgh \frac{u_x}{2\varphi_T} \approx \frac{u_x}{2\varphi_T} \quad \text{for} \quad |u_x| << 2\varphi_T$$

$$u_{out} = (I_0 + g_m u_y) R_C tgh \frac{u_x}{2\varphi_T} \approx I_0 R_C \frac{u_x}{2\varphi_T} + g_m R_C \frac{u_x u_y}{2\varphi_T}$$

Linear range of input voltage

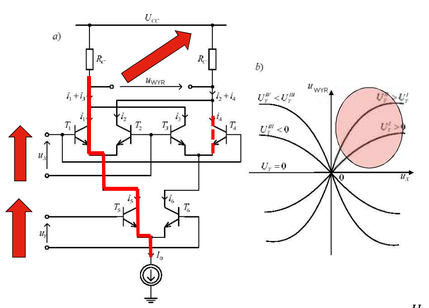


4-quadrant multiplier



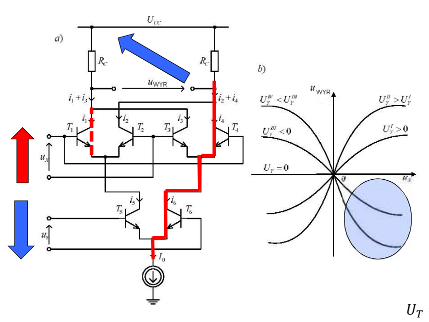
$U_T = \varphi_T$

4-quadrant multiplier



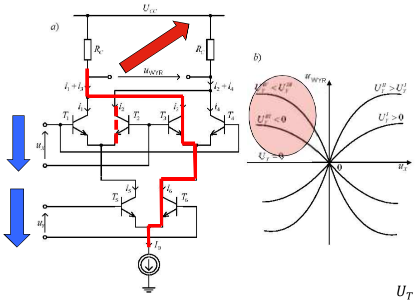
$U_T = \varphi_T$

4-quadrant multiplier



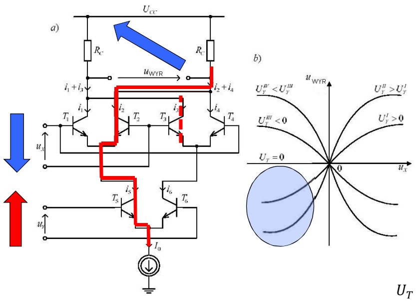
$U_T = \varphi_T$

4-quadrant multiplier



$U_T = \varphi_T$

4-quadrant multiplier



$U_T = \varphi_T$

4-quadrant multiplier

$$i_{out} = i_5 tgh\left(\frac{u_x}{2\varphi_T}\right) - i_6 tgh\left(\frac{u_x}{2\varphi_T}\right) = \left(i_5 - i_6\right) tgh\left(\frac{u_x}{2\varphi_T}\right) =$$

$$= I_0\left(\frac{u_y}{2\varphi_T}\right)\left(\frac{u_x}{2\varphi_T}\right) \approx \frac{I_0}{4\varphi_T^2} u_x u_y$$

for: $|u_x|, |u_y| << \varphi_T$

Napięcie wyjściowe układu dane jest zależnością: $u_{out} = i_{out} R_C$

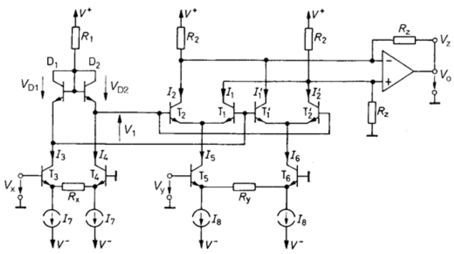
4-quadrant multiplier linearization

Very small input voltage range

Can be eleminated by:

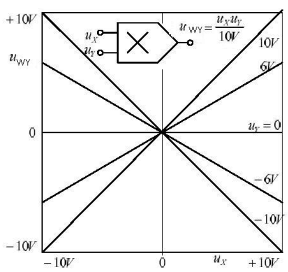
- Gilbert cell,
- additional R_E

4-quadrant multiplier with Gilbert cell

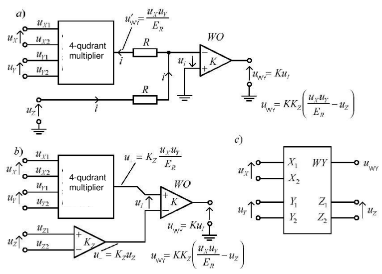


Tietze, Schenk

4-quadrant multiplier after linearisation



4-quadrant multiplier analog IC examples



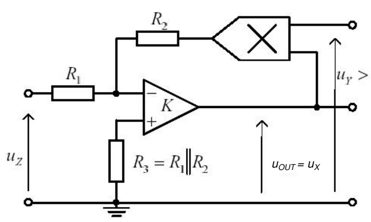
Analog IC multipliers

IC Type	Manufacturer	Accuracy		Bandwidth	
		without adjustment	with adjustment	1%	3 dB
MPY 100	Burr Brown	0.5 %	0.35%	35 kHz	0.5 MHz
MPY 600	Burr Brown	1 %	0.5 %		60 MHz
AD 534	Analog Dev.	0.25%	0.1 %	70 kHz	1 MHz
AD 633	Analog Dev.	1 %	0.1 %	100 kHz	1 MHz
AD 734	Analog Dev.	0.1 %		1000 kHz	10 MHz
AD 834	Analog Dev.	2 %			500 MHz
AD 835	Analog Dev.		0.1 %	15 MHz	250 MHz
MLT 04*	Analog Dev.	2 %	0.2 %		8 MHz

* 4 multipliers on 1 chip

Tietze, Schenk

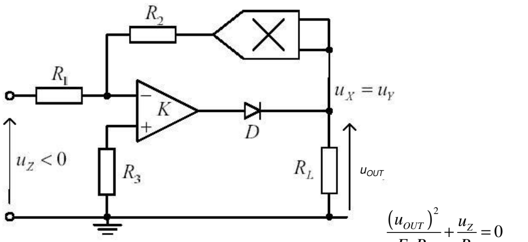
Analog divider



$$\frac{u_{OUT} u_Y}{E_R R_2} + \frac{u_Z}{R_1} = 0$$

$$u_{OUT} = -\frac{u_Z}{u_Y} \frac{R_2}{R_1} E_R$$

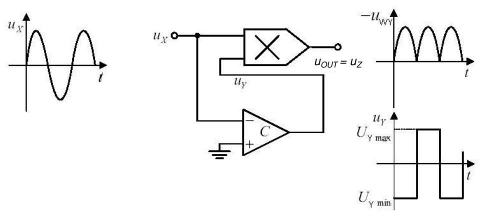
Analog squerer



$$\frac{(u_{OUT})^2}{E_R R_2} + \frac{u_Z}{R_1} = 0$$

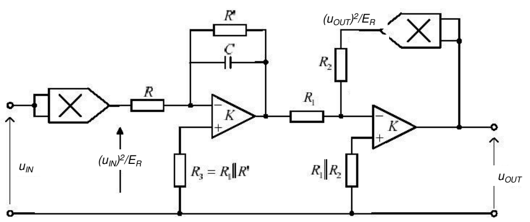
$$u_{OUT} = \sqrt{-u_Z \frac{R_2}{R_1} E_R}$$

Abs function („prefect" rectifier)



$$u_{OUT} = u_X \operatorname{sgn}(u_X)$$

Analog true RMS transducer



$$u_{OUT} = \sqrt{\frac{R_2}{R_1}\frac{1}{RC}\int_0^T u_{IN}^2(t)\,dt}$$

Triangle - sin transducer

Triangle:

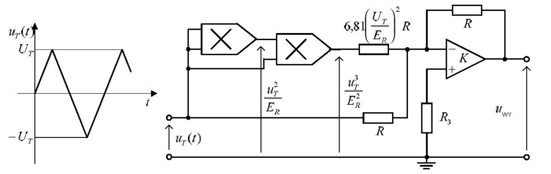
$$u_T(t) = (-1)^k \frac{2U_T}{\pi}(\omega t - k\pi) \text{ dla } \frac{(2k-1)\pi}{2} \le \omega t \le \frac{(2k+1)\pi}{2}$$

SIN Polynomial approximation:

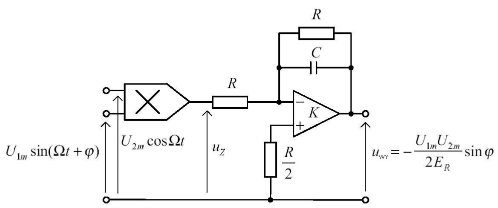
$$U_m \sin \omega t = U_m \left[\omega t - \frac{(\omega t)^3}{3!} + \frac{(\omega t)^5}{5!} - \ldots \right]$$

$$u_{OUT} = U_m \sin \omega t \approx U_m \left[\omega t - \frac{(\omega t)^3}{6.81} \right]$$

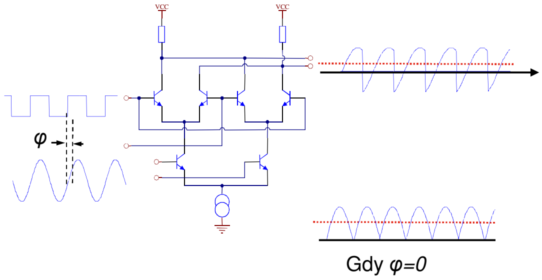
Triangle - sin transducer



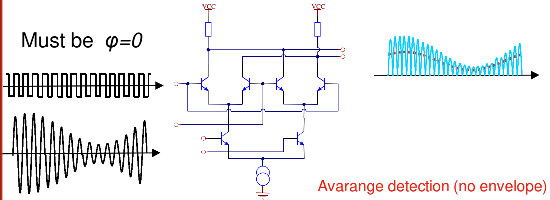
Phase detector



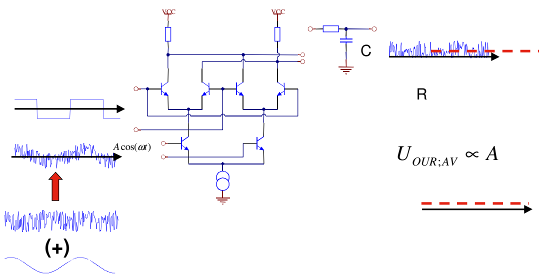
Small signal phase detector



AM synchronous demodulator



Synchronous detection lock-in-amplifier



Summary

1. Types of analog multipliers
   - Pulse modulation
   - Log/Exp circuits
   - With squarer
   - Transconductance (2-quadrant; 4-quadrant)
2. Applications
   - divider
   - Square root
   - Rectifier (abs)
   - RMS transducer
   - Triangle – sin transducer
   - Phase detector (to be continued)

Wroclaw University of Science and Technology

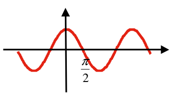
Synchronous detection & PLL

Synchronous detection

For

$U_1 cos(\omega_1 t)$ and $U_2 cos(\omega_2 t+\varphi)$ and $U_2 >> \varphi_T$

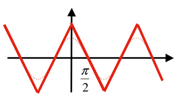
$$U_{OUT,AV} = \begin{cases} U_1 \cos\phi & dla \quad \omega_1 = \omega_2 \\ 0 & dla \quad \omega_1 \neq \omega_2 \end{cases}$$



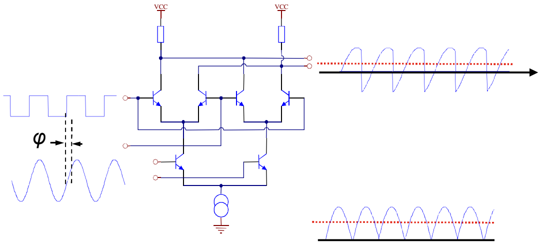
For

$U_1 rec(\omega_1 t)$ and $U_2 rec(\omega_2 t)$

$$U_{OUT,AV} = \begin{cases} U_1(\phi - \pi/2) & dla \quad \omega_1 = \omega_2 \\ 0 & dla \quad \omega_1 \neq \omega_2 \end{cases}$$

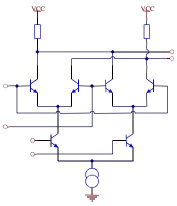


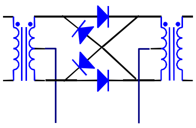
Small signal phase detector



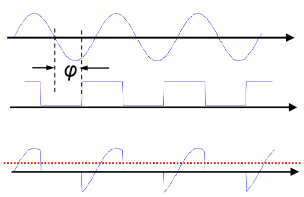
Gdy $\varphi=0$

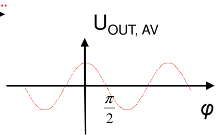
Analog multiplier = phase detector



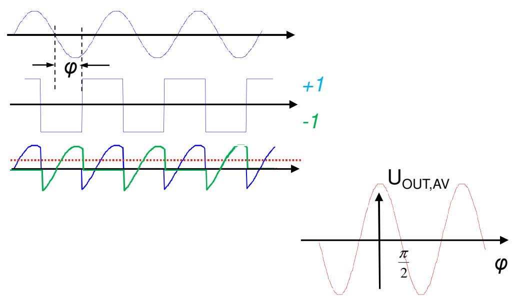


2-quadrant multiplication

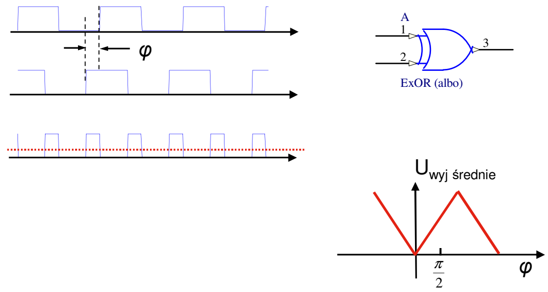




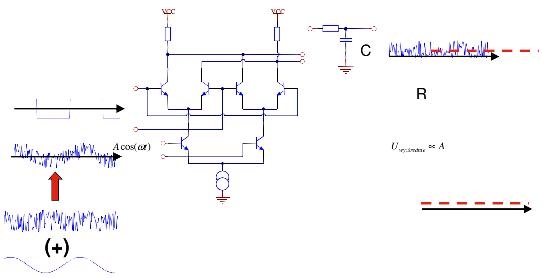
4-quadrant multiplication



Digital multiplexing - analog output

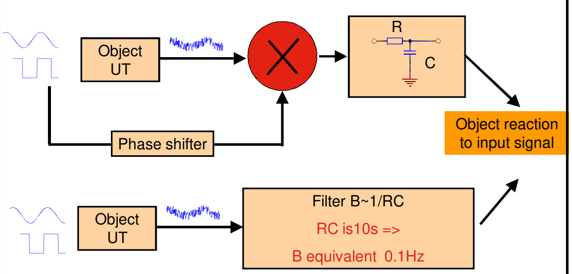


Synchronous detection



Synchronous detection

Multiplier == synchronous detection

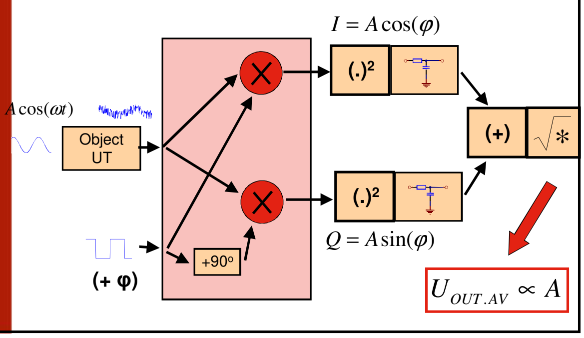


Synchronous detection

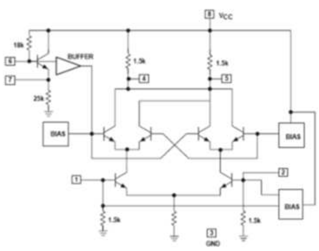
- Synchronous detection
- Homodyne detection
- Lock-in amplifier

Lock-in amplifier example

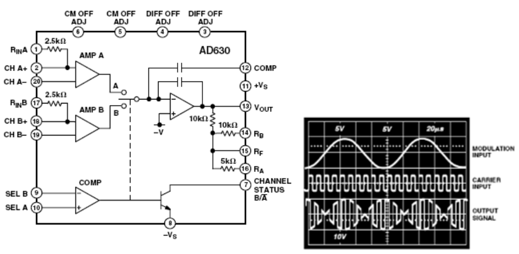
Synchonous detection even if φ≠0
„quadrature detection” „I&Q detector”



Quadrature detector
(also mixer)



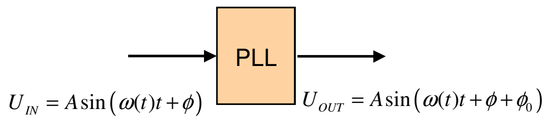
Synchronous detector example: AD630



Modulacja DSB-S.C. !!!

PLL (Phase Locked-Loop)

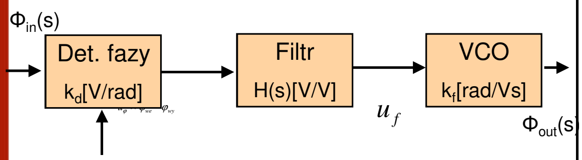
Regenerate signal of „exactly" same frequency and phase (shift) !!!!!!



PLL

- Synchronize to fundamental frequency or (not always) to harmonics,
- Is able to keep generation even if input signal disappear,
- Input signal can be disturbed(in phase or amplitude),
- Can extract signal from noise (like selective filter)

PLL - block diagram



„Phase signal”

$$U_{in} = A\sin\left(\omega t + \Phi_{in}(t)\right)$$
$$\Phi_{in}(t) \succ \Phi_{in}(s)$$
$$U_{out} = A\sin\left(\omega t + \Phi_{out}(t)\right)$$
$$\Phi_{out}(t) \succ \Phi_{out}(s)$$

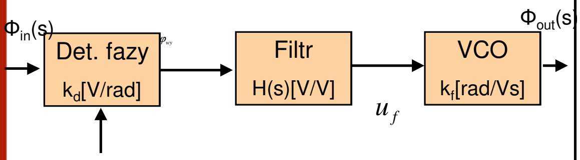
VCO as integrater

$$A\sin\left(\omega_0 t + \varphi(t)\right) = A\sin\left(\Phi(t)\right)$$
$$\omega(t) \equiv \frac{d\Phi(t)}{dt} \quad z \quad definicji$$
$$\omega(t) \equiv \omega_0 + k_f u(t) \quad z \quad \text{"} potrzeby \text{"}$$
$$\frac{d\Phi(t)}{dt} = \omega_0 + k_f u(t)$$
$$\Phi(t) = \int_0^t \left(\omega_0 + k_f u(t)\right) dt = \omega_0 t + k_f \int_0^t u(t) dt$$
$$\varphi(t) \succ \frac{k_f}{s} U(s)$$

In Laplace transformation integration corresponds to division by s

PLL - diagram

$$G_{otw}(s) \equiv \frac{\Phi_{out}(s)}{\Phi_{in}(s)} = k_d H(s) \frac{k_f}{s}$$

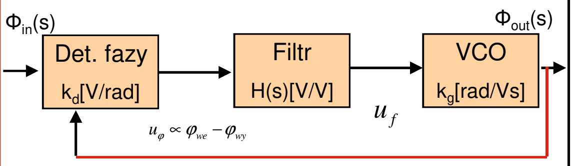


Typ of PLL system

$$G_{OpenLoop}(s) \equiv \frac{\Phi_{out}}{\Phi_{in}} = k_d \frac{k_f}{s} H(s) = \frac{numerator(s)}{s^{type}(polynomial(s))}$$

Type = the multiplicity of zero pole of transferr function with open loop

PLL - block diagram

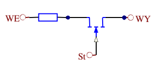


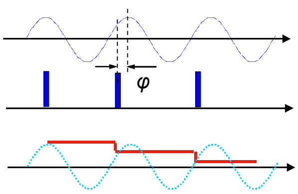
$$G_{zam}(s) \equiv \frac{\Phi_{out}}{\Phi_{in}} = \frac{k_d k_g H(s)}{s + k_d k_g H(s)} = \frac{numerator(s)}{denominator(s)} = \frac{numerator(s)}{s^{order} + rest(s)}$$

Order of PLL=

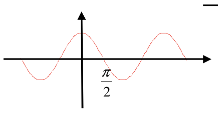
Order of denominator of tr.fuc. with closed loop

Phase detector sample-hold circuit

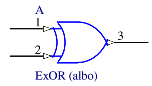


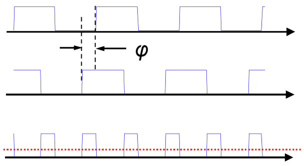


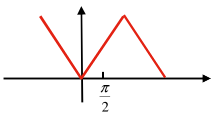
$$U_{IN;AV} = \frac{U_{0IN}}{\pi}\cos(\phi)$$



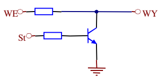
Phase detector
EXOR gate



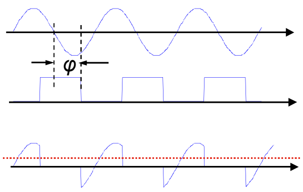




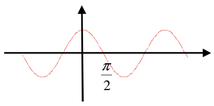
Phase detector
switch (multiplier)



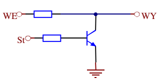
$$U_{IN} = U_{0IN}\sin(\omega t)$$

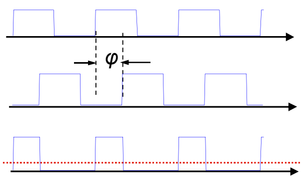


$$U_{OUT;AV} = \frac{U_{0IN}}{\pi}\cos(\phi)$$



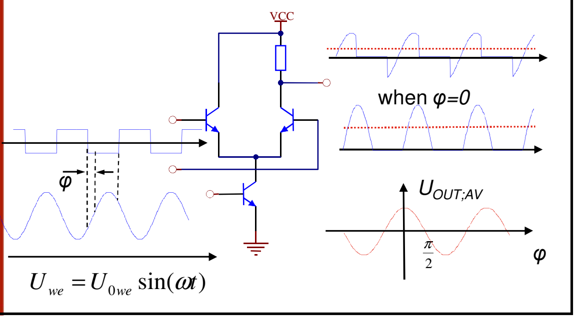
Phase detektor
switch (multiplier)



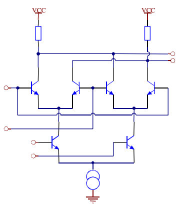


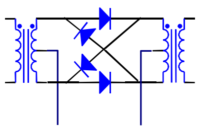
$$U_{OUT,AV} = U_{0IN}\phi / 2$$

Phase detektor
analog multiplier

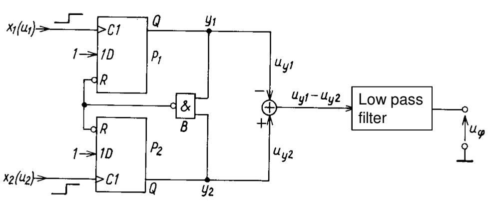


Double-balanced mixer

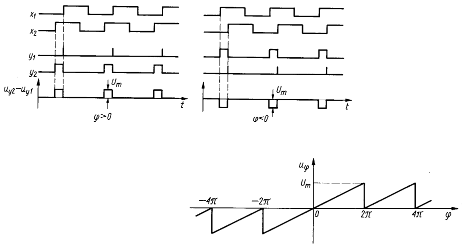




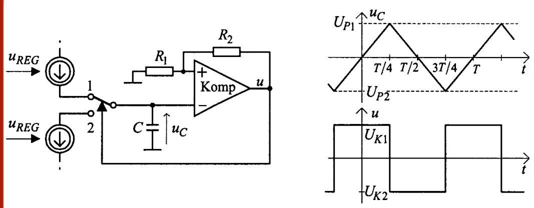
Phase and frequency detector
D flip-fop



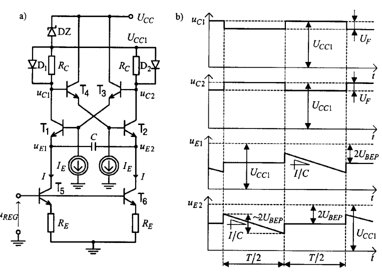
Phase and frequency detector D flip-fop



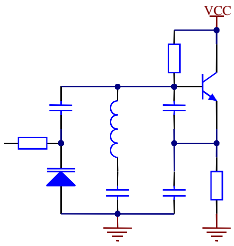
VCO - integrator + flip-flop



VCO - emitter coupled multivibrator



Direct modulation
VCO- Voltage Controlled Oscillator



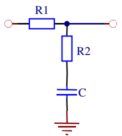
$$\Omega = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{C_o}{\sqrt{1 - {u}/{V}}}$$

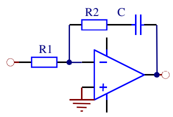
$$\Omega = \Omega_0 + f(m(t))$$

Nonlinear function !!!!!

PLL = LP filter

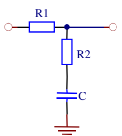


$$H(s) = \frac{1 + R_2 Cs}{1 + (R_1 + R_2)Cs}$$



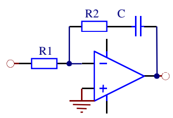
$$H(s) = \frac{1 + R_2 Cs}{R_1 Cs}$$

PLL funkcja przenoszenia układu z pętlą otwartą



$$G_{otw}(s) = \frac{1 + R_2 Cs}{1 + (R_1 + R_2)Cs} \cdot \frac{k_d k_f}{s}$$

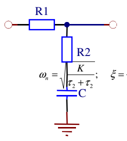
Type one



$$G_{otw}(s) = \frac{1 + R_2 Cs}{R_1 Cs} \cdot \frac{k_g k_f}{s}$$

Type second

PLL close loop transfer function



$\omega_n = \sqrt{\frac{K}{\tau_2 + \tau_2}}; \quad \xi = \frac{1}{2}\omega_n\left(\frac{1 + K\tau_2}{K}\right)$

$\tau_1 = R_1 C \quad \tau_2 = R_2 C \quad K = k_d k_f$

$$G_{zam}(s) = \frac{\omega_n\left(2\xi - \frac{\omega_n}{K}\right)s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

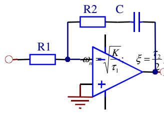
Second order

$$\Delta\omega_L = 2\xi\omega_n$$

$$\Delta\omega_C = \frac{8}{\pi}\sqrt{K\xi\omega_n - \omega_n^2}$$

$$t_C = \frac{(\Delta\omega)^2}{2\xi\omega_n^2}$$

PLL PLL close loop transfer function



$\omega_n = \sqrt{\frac{K}{\tau_1}}; \quad \xi = \frac{\tau_2}{2}\sqrt{\frac{K}{\tau_1}}$

$\tau_1 = R_1 C \quad \tau_2 = R_2 C \quad K = k_d k_f$

$$G_{zam}(s) = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Second order

$$\Delta\omega_L = 2\xi\omega_n$$

$$\Delta\omega_C = \frac{8}{\pi}\sqrt{K\xi\omega_n - \omega_n^2}$$

$$t_C = \frac{(\Delta\omega)^2}{2\xi\omega_n^2}$$

PLL - response to input changes

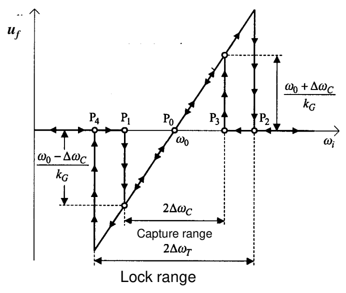
$$\Phi_{out}(s) = G_{zam}(s)\Phi_{in}(s)$$

$$\Phi_{out}(t) \succ \Phi_{out}(s)$$

$$\omega(t) = \frac{d\Phi_{out}(t)}{dt}$$

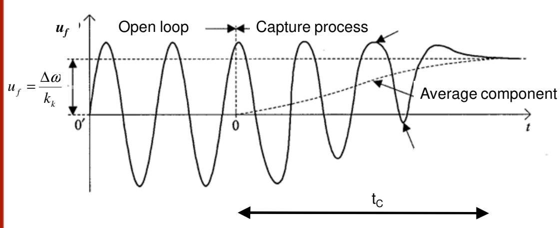
Skokowa zmiana fazy (np.. PSK impulsowa modulacja fazy)	$\Phi_{in}(t) = \Delta\varphi$	$\Phi_{in}(s) = \frac{\Delta\varphi}{s}$
Skokowa zmiana częstotliwości (np. FSK impulsowa modulacja częstotliwości)	$\Phi_{in}(t) = \Delta\omega t$	$\Phi_{in}(s) = \frac{\Delta\omega}{s^2}$
Liniowa zmiana częstotliwości (np. modulacja „chirp”)	$\Phi_{in}(t) = \Delta\omega t^2$	$\Phi_{in}(s) = \frac{V}{s^3}$

PLL capture and lock ranges



tu $k_G = k_f$

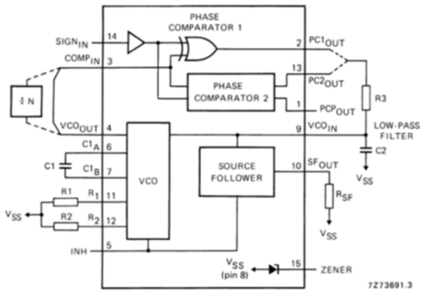
PLL synchronization process



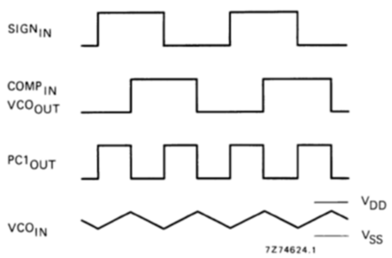
PLL applications

- AM demodulation
- Modulation and demodulation of FM i PM
- Frequency synthesis
- Synchonous detection (reference clock regeneration)
- Telekomunication (clock regeneration)

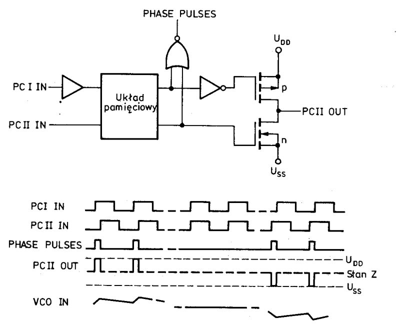
IC 4046



4046 Phase detector I



4046 phase detector II



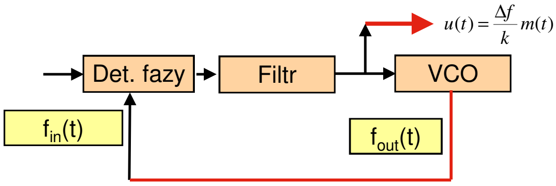
4046 features

Characteristic	Using Phase Comparator 1	Using Phase Comparator 2
No signal on input PCA_{in}.	VCO in PLL system adjusts to center frequency (f_0).	VCO in PLL system adjusts to minimum frequency (f_{min}).
Phase angle between PCA_{in} and PCB_{in}.	90° at center frequency (f_0), approaching 0° and 180° at ends of lock range ($2f_L$)	Always 0° in lock (positive rising edges).
Locks on harmonics of center frequency.	Yes	No
Signal input noise rejection.	High	Low
Lock frequency range ($2f_L$).	The frequency range of the input signal on which the loop will stay locked if it was initially in lock; $2f_L$ = full VCO frequency range = $f_{max} - f_{min}$.	
Capture frequency range ($2f_C$).	The frequency range of the input signal on which the loop will lock if it was initially out of lock.	
	Depends on low−pass filter characteristics (see Figure 3). $f_C \le f_L$	$f_C = f_L$

4046 features

Charakterystyka	Detektor I (efor)	Detektor II
Brak sygnału na wejściu	$f_{wyj}=f_0$	$f_{wyj}=f_{min}$
Przesunięcie fazy we/wy	90deg dla f_0 0 do 180 na granicach $2f_T$	0deg
Syn. Do harmonicznych	synchronizuje	Nie synchronizuje
Odporność na szum	duza	mała
$2f_T$ (trzymanie)	$f_{max} - f_{min}$	
$2f_C$ (chwytanie)	$f_C < f_T$ (zależy od filtru)	$f_C = f_T$

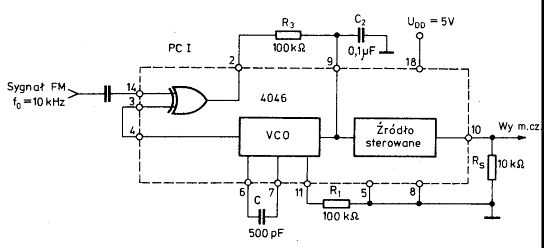
FM demodulator



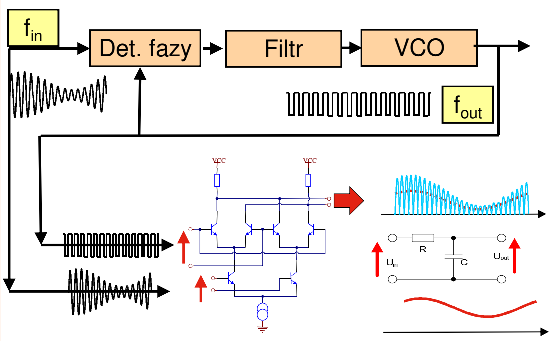
$$f_{in}(t) = f_0 + m(t) \cdot \Delta f$$
$$f_{out}(t) = f_0 + m(t) \cdot \Delta f$$
$$f_{out}(t) = f_0 + u(t) \cdot k$$

$$u(t) = \frac{\Delta f}{k} m(t)$$

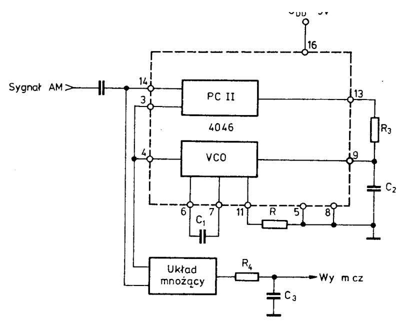
4046 demodulator FM



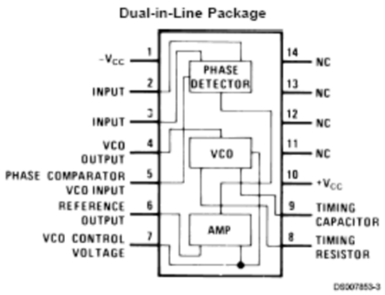
AM synchronous detection



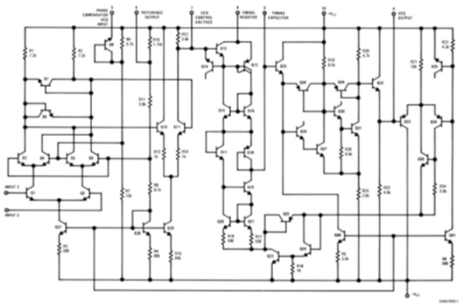
4046 AM demodulator



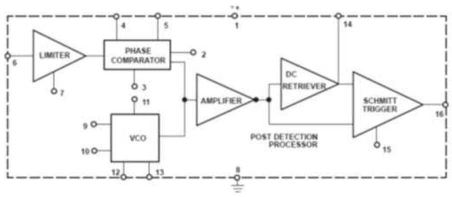
PLL IC
LM565 (do 500kHz)



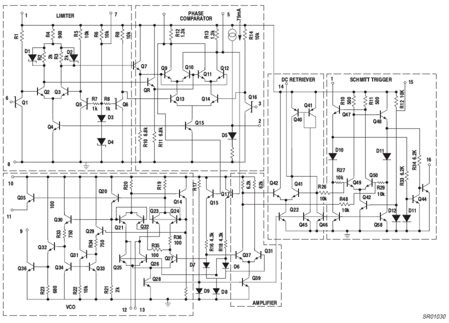
LM565



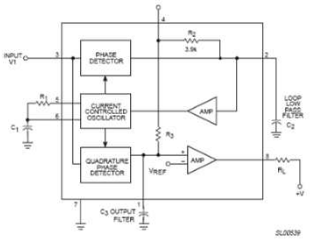
NE564



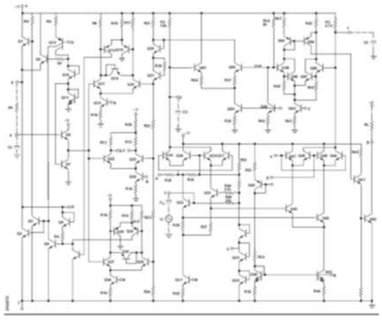
NE564 (up to 50MHz)



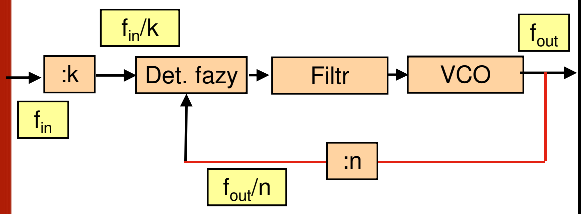
NE567



NE567

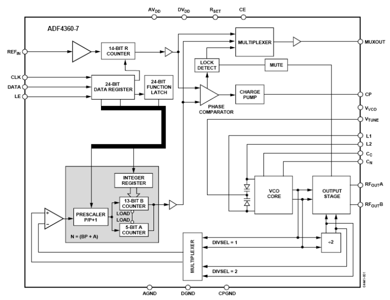


Frequency synthesizer

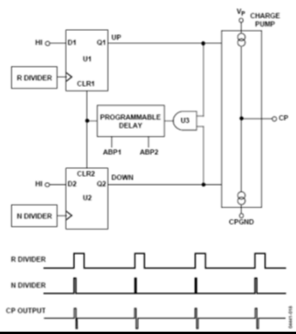


$$\frac{f_{in}}{k} = \frac{f_{out}}{n} \quad \Rightarrow \quad f_{out} = \frac{n}{k} f_{in}$$

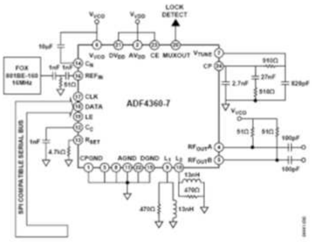
ADF4360-7 (350-1800MHz)



ADF4360-7 (350-1800MHz)



ADF4360-7 generater 500MHz



Summary synchronous detection and PLL

- Idea of synchronous detection
- Double balanced mixer
- I & Q detector
- Examples of (phase detector, VCO, filter)
- PLL principle
- What are lock and capture frequency ranges ?
- Applications (AM detector, FM detector, frequency synthesizer)